NAG Toolbox for MATLAB

s19ad

1 Purpose

s19ad returns a value for the Kelvin function $\mathrm{kei}\, x$ via the function name.

2 Syntax

```
[result, ifail] = s19ad(x)
```

3 Description

s19ad evaluates an approximation to the Kelvin function $\mathrm{kei}\, x$.

Note: for $x < 0$ the function is undefined, so we need only consider $x \geq 0$.

The function is based on several Chebyshev expansions:

For $0 \leq x \leq 1$,

$$\mathrm{kei}\, x = -\frac{\pi}{4} f(t) + \frac{x^2}{4}[-g(t) \log x + v(t)]$$

where $f(t)$, $g(t)$ and $v(t)$ are expansions in the variable $t = 2x^4 - 1$;

For $1 < x \leq 3$,

$$\mathrm{kei}\, x = \exp\left(-\frac{9}{8}x\right) u(t)$$

where $u(t)$ is an expansion in the variable $t = x - 2$;

For $x > 3$,

$$\mathrm{kei}\, x = \sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}} \left[\left(1 + \frac{1}{x}\right) c(t) \sin\beta + \frac{1}{x} d(t) \cos\beta\right]$$

where $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$, and $c(t)$ and $d(t)$ are expansions in the variable $t = \frac{6}{x} - 1$.

For $x < 0$, the function is undefined, and hence the function fails and returns zero.

When x is sufficiently close to zero, the result is computed as

$$\mathrm{kei}\, x = -\frac{\pi}{4} + \left(1 - \gamma - \log\left(\frac{x}{2}\right)\right) \frac{x^2}{4}$$

and when x is even closer to zero simply as

$$\mathrm{kei}\, x = -\frac{\pi}{4}.$$

For large x, $\mathrm{kei}\, x$ is asymptotically given by $\sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}}$ and this becomes so small that it cannot be computed without underflow and the function fails.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $\mathbf{x} \geq 0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **x** is too large: the result underflows. On soft failure, the function returns zero.

ifail $= 2$

On entry, $\mathbf{x} < 0$: the function is undefined. On soft failure the function returns zero.

7 Accuracy

Let E be the absolute error in the result, and δ be the relative error in the argument. If δ is somewhat larger than the machine representation error, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (-\mathrm{ker}_1 x + \mathrm{kei}_1 x) \right| \delta.$$

For small x, errors are attenuated by the function and hence are limited by the ***machine precision***.

For medium and large x, the error behaviour, like the function itself, is oscillatory and hence only absolute accuracy of the function can be maintained. For this range of x, the amplitude of the absolute error decays like $\sqrt{\frac{\pi x}{2}} e^{-x/\sqrt{2}}$, which implies a strong attenuation of error. Eventually, $\mathrm{kei}\, x$, which is asymptotically given by $\sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}}$,becomes so small that it cannot be calculated without causing underflow and therefore the function returns zero. Note that for large x, the errors are dominated by those of the standard function EXP.

8 Further Comments

Underflow may occur for a few values of x close to the zeros of kei x, below the limit which causes a failure with **ifail** $= 1$.

9 Example

```
x = 0;
[result, ifail] = s19ad(x)
```

```
result =
   -0.7854
ifail =
           0
```